Collection Efficiency Model Based on Boundary-Layer Characteristics for Cyclones

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In cyclones, the boundary layer formed on the collecting-wall surface acts as a barrier for particle migration toward the wall due to a decreased centrifugal force on particles inside the boundary layer. A new theory for high-efficiency cyclones based on the boundary-layer characteristics is presented. The cyclone was divided into two regions: the turbulent-core region where the centrifugal force is large, and the near-wall region where the centrifugal force is small. Particle trajectories in the turbulent-core region are calculated from the mean fluid motion based on the quasi-steady drag assumption, and the collection probability of particles in the near-wall region is calculated by the deposition velocity that results from both turbulent diffusion and centrifugal force. The deposition velocity by centrifugal force was assumed equal to the equilibrium migration velocity at a certain point inside the boundary layer, and the distance to that point from the wall is assumed to be linearly proportional to the dimensionless-particle relaxation time. When the proportional constant was determined by fitting the theoretical results to experimental data, the theory showed an excellent enhancement in predicting the variation of collection efficiency with the inlet flow velocity and particle size.

Introduction

In a cyclone a steady swirling flow is maintained and each particle suspended in the swirling flow experiences a centrifugal force acting radially outward. The centrifugal force induces a radial migration of particles, and the migration velocity is established by a balance between the centrifugal force and the hydrodynamic drag force. Since the centrifugal force acting on a particle is proportional to the circumferential velocity squared and the hydrodynamic drag is dependent on the relative velocity between gas flow and the particle, an accurate estimation of radial migration velocity requires an accurate gas velocity distribution, particularly when the velocity is not uniform in the cyclone.

Of various types of cyclones, the reverse-flow-type cyclone with tangential inlet (Figure 1) is used most often. This is also the type of cyclone considered in this article. The gas flow field in a reverse-flow-type cyclone is normally divided into two regions, the radially outer region and the inner region. In the outer region (annular region) the gas stream swirls down and has a radially inward velocity component. In the inner region (vortex core region) the gas flows upward with a strong swirl toward the outlet (vortex finder). The interface between the inner region and the outer region is roughly defined as an extension of the outlet tube. The profile of the tangential velocity, which is the major driving force for the separation of particles, is different in those two regions. While the tangential velocity increases with radius R in the vortex core region, it decreases in the annular region as R^{-n} with $n = 0.5 \sim 0.9$. This power-law decrease of tangential velocity is maintained down to the edge of the thin boundary layer, inside which the velocity decreases rapidly to zero on the wall.

On entering a cyclone, particles begin to migrate toward the wall, with some dispersion due to turbulence. And the probability of a particle being collected on the wall will be determined by the ratio of two time scales; the migration time, which is the time required to reach the wall by radial migration, and the flow time, which is the residence time available for migration. The flow time is usually defined in terms of cyclone height and mean axial flow velocity, and the migration time in terms of the inlet half-width and the mean migration velocity. In an exact sense, the migration time de-

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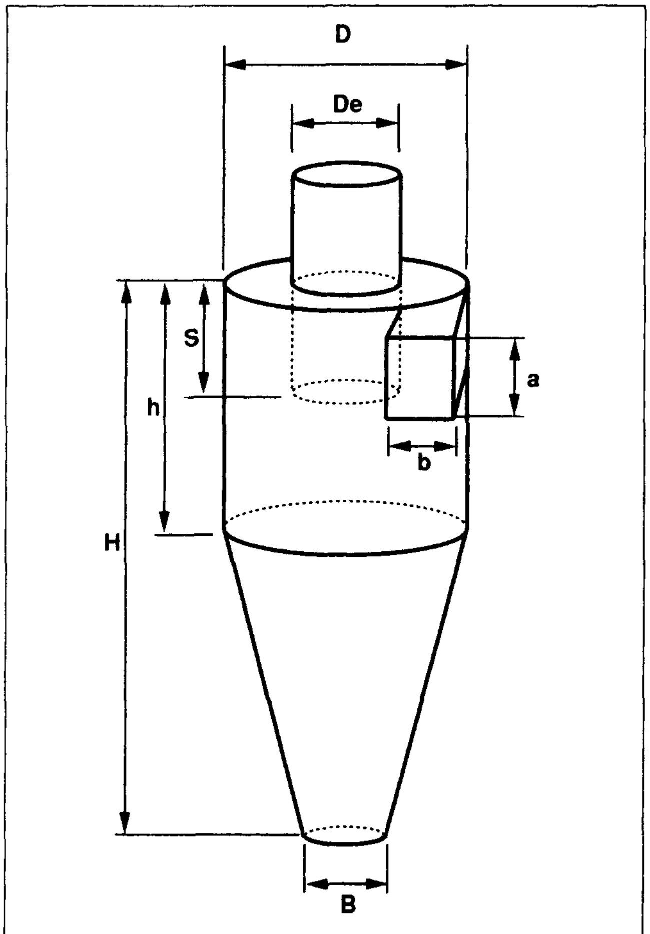

Figure 1. Definition of geometric dimensions of a reversed-flow cyclone.

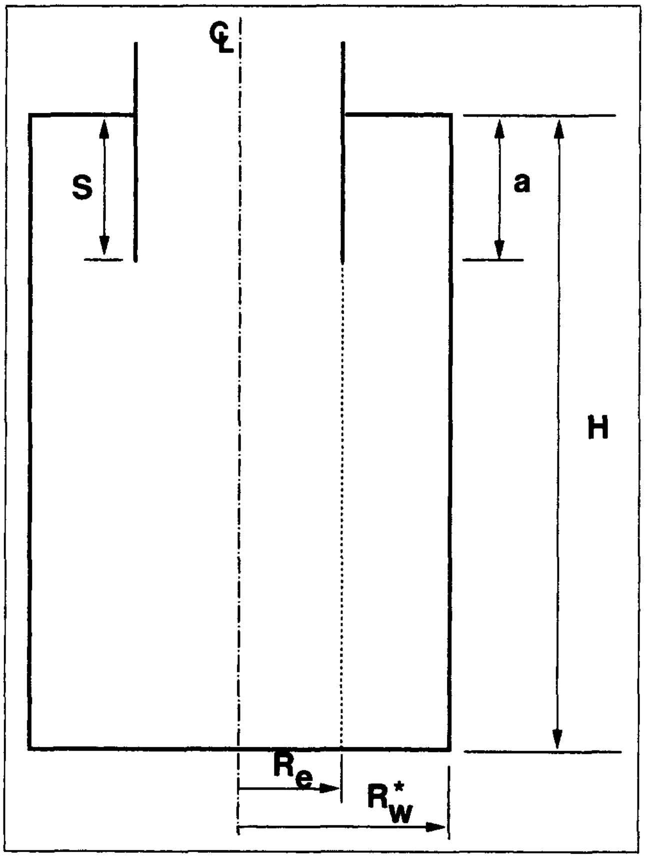

Figure 2. Modified cyclone geometry for analysis by Mothes and Löffler (1984b).

fined as such represents the time required to reach the near-wall region, the so-called boundary layer, not the wall surface, but all the existing theories are based on that migration time. The validity of that migration time for estimating collection efficiency needs to be examined, which is the objective of this study. A general review of the existing theories is given first.

Approaches to predict the collection efficiency of cyclones have been divided into critical-diameter approaches and fractional-efficiency approaches (Leith, 1979). In the critical diameter approaches, theories are developed to predict just the cut diameter d_{50}, which corresponds to the particle size whose collection efficiency is 50% or the terminal settling velocity for the static particles v_{ts}^*. Collection efficiencies for other particle sizes are found from a generalized efficiency curve, where the efficiency for particles of a certain size are expressed as a function of the ratio of particle diameter to the cut diameter, d_p/d_{50} (Lapple, 1950), or the ratio of terminal settling velocity of a particle to that of the static particle, v_{ts}/v_{ts}^* (Barth, 1956). Since the generalized efficiency curve comes from experimental data fitted to presumed functional forms, the results of a particular critical-diameter approach may not hold good for other conditions that are used in formulating the theory.

The fractional efficiency approaches try to directly calculate collection efficiencies for particles of any size and for cyclones of any design, based on the conservation principle of particles with some simplifying assumptions for cyclone geometry, flow field, and particle motion.

In the Leith–Licht model (1972), uncollected dusts are assumed completely and uniformly mixed by turbulence at any height in a cyclone, and the particle-collection flux is calculated in terms of the residence time of particles in the cyclone, with deterministically calculated particle trajectories. The assumption of deterministic trajectories is contradictory to the assumption of complete mixing. On noticing the inconsistency, Clift et al. (1991) corrected for the particle removal rate on the wall, as in Dietz (1981), by using the particle migration velocity at the wall and the mean concentration of particles under the same mixing condition as in the Leith–Licht model. They also used the flow residence time of Danckwerts (1953) that regardless of the configuration of the inlet or outlet mean flow residence time in a steady flow system is given by the system volume divided by the volumetric throughput, which implies that Leith and Licht underestimated the mean flow residence time because they took the inside volume to be much smaller considering the distributed flow characteristics at inlet and interface between outer and inner vortex region.

Dietz (1981) divided a cyclone into three regions considering the reverse flow characteristics: the entrance region, the downflow (or annular) region, and the upflow (or core) region. He also assumed that turbulence produces a radially uniform concentration of uncollected particles at any height, but only within each region. Mothes and Löffler (1984b) modified the Dietz theory with respect to the particle exchange between the core region and the annular region by introducing an effective particle dispersion coefficient, D_p, which is generally unknown.

In these fractional efficiency approaches, the collection flux on the wall is usually given by the particle concentration multiplied by the radial migration velocity of each particle size. The radial migration velocity is calculated from the particle tangential velocity, which is assumed equal to the gas velocity at the turbulent core edge above the wall boundary layer.

Enliang and Yingmin (1989) proposed a new cyclone theory in which the turbulent particle dispersion is neglected throughout the interior region of a cyclone, even with a particle concentration gradient maintained, and at the same time, the particle concentration gradient at the wall is assumed zero, but with a finite turbulent diffusivity at the wall. Although their results show good agreement with the experimental data of Dirgo and Leith (1985), their theory is questionable in view of an abnormally high tangential velocity, unrealistic wall boundary condition, and the violated conservation of particles.

A successful model should predict the change in collection efficiency for changes in various important parameters, typical of which are the cyclone geometry, gas velocity at the inlet, and the particle diameter. The effect of the cyclone geometry—the body, the inlet, and the outlet—is the hardest part to be considered accurately because the characteristics of the gas flow field change much with the geometry. However, once the geometry is fixed, the general features of the flow field remain even when gas velocity is varied, so a good model should be able to predict the change in efficiency with changing gas velocity. When the geometry is fixed, the collection efficiency predicted by theories is dependent on the operational conditions such as the inlet velocity and particle size. In most of the previous models, the estimated collection efficiency becomes a function of only one variable, inlet gas velocity times particle diameter squared, which corresponds to the ratio of radial migration velocity to streamwise velocity in the turbulent core. On the other hand, the experimental results show that the grade efficiency cannot be well described by the ratio of velocities in the turbulent core alone, but that other factors have to be introduced.

When particles migrate close to the collecting wall to enter the boundary layer, the gas velocity decreases very rapidly; consequently, the particle residence time available for migration (flow time) increases. If the migration velocity to the wall is not changed as in the parallel duct flow with constant body force (electrostatic precipitators, for example), the boundary layer gives particles more time to be collected, and then it becomes possible to assume that all the particles entering the boundary layer are collected. If this is the case, the probability of reaching the boundary layer is almost the same as that of reaching the wall, and we have only to analyze the bulk-flow region, assuming that the bulk-flow region extends to the wall. However, in a cyclone the migration velocity decreases much more rapidly than the flow velocity when approaching the wall because the centrifugal force changes as a second power to the tangential velocity. Hence the required migration time increases much more rapidly than the available flow residence time, and as a result the boundary layer acts as a barrier for particles to be collected on the wall. Furthermore, since the equilibrium (or static) migration velocity becomes zero just above the wall surface, actual deposition should result from a dynamic penetration through the boundary layer, for which a theory based on quasi-steady migration would fail.

It thus follows that a new cyclone efficiency model is necessary where the effects of wall boundary layer on particle deposition is taken into account.

Modeling Cyclones Based on Boundary-Layer Characteristics

In cyclones, the driving force that separates particles from the gas stream is the centrifugal force, which also induces a nonzero concentration gradient for large particles (Mothes and Löffler, 1982). Whereas the centrifugal force is large in the turbulent-core region due to the high swirling velocity, it decreases rapidly with gas velocity as particles migrate toward the wall and enter the boundary layer. On the other hand, flow turbulence makes particles disperse and plays another role in particle deposition on the wall. So the effects of the boundary layer and turbulent dispersion can be seen, a cyclone is divided into two regions; the turbulent-core region, where the centrifugal force is large, and the near-wall region, where the centrifugal migration is small.

In the turbulent core region, the flow velocity changes quite slowly in the radial direction, so the "mean" particle trajectories can be predicted quite accurately based on the assumption that the particle migration velocity at each position is the steady migration velocity corresponding to the local gas velocity (quasi-steady drag assumption). Therefore, though the turbulent intensity is very high in the turbulent-core region, the "average particle flux" into the boundary layer can be predicted quite accurately no matter whether the turbulent dispersion is considered or not.

On the other hand, in the near-wall region where the boundary layer effects are considered, the flow velocity changes very abruptly, so the particle motion cannot become equilibrated with the surrounding gas velocity. In such a situation, the particle migration velocity cannot be determined from the local conditions only, but becomes dependent on the past history. Even though the particle motion in the boundary layer is dynamic, all the average transport characteristics resulting from dynamic particle motion can be assumed constant once the flow field in the turbulent core and in the boundary layer are fully developed. So, in this study the flow fields are assumed fully developed throughout the cylone, then the boundary-layer thickness is uniform over the entire wall, and the so-called deposition velocity on the wall is also uniformly constant. The real deposition velocity can be estimated only by the Lagrangian analysis, but instead a simple model is introduced in this study that the penetration depth rather than the deposition velocity is proportional to the particle inertia, and the proportional constant is sought by comparing with the experimental data. Then the complicated motion of particles in the boundary layer is treated by using average parameters—the average flow velocity, the average boundary-layer thickness, the average deposition velocity, and the average concentration.

As to the average concentration at different positions in the boundary layer, the trajectories of particles in the turbulent-core region are calculated by a Lagrangian integration until they arrive at the near-wall region, and from these trajectories the particle flux distribution over the boundary-layer surface can be deduced. Then the average concentration at each section of the boundary layer can be easily calculated once the deposition velocity is known.

Model geometry

In developing theories, the conical shape of a real cyclone is usually replaced by an equivalent right circular cylinder in order to simplify the analysis. There are two different ways of obtaining an equivalent cylinder: one is to keep the real radius but adjust the height, and the other is to keep the real height and to adjust the radius. Leith and Licht (1972), Dietz (1981), and Clift et al. (1991) modified the height using the so-called "natural length," which was defined experimentally by Alexander (1949) as the farthest distance the spinning gas extends below the bottom of the vortex finder. Mothes and Löffler (1984b) modified the radius rather than the height (Figure 2).

Whether to modify the radius or the height requires some careful consideration. In the conical section of real cyclones, the tangential velocity profile tends to maintain its shape and magnitude at each cross section in spite of the change in cross-sectional area (Boysan et al., 1983). Therefore the near-wall tangential velocity becomes larger in the conical section than in the cylindrical part of a cyclone.

Considering that the average migration velocity is higher and most of the collection occurs in the conical section, it seems more reasonable to modify the radius than the height in such a way that the effective migration velocity near the wall and mean flow residence time in an equivalent cylinder become the same as those in the real cyclone, as in Mothes and Löffler (1984b). The modified radius, R_w^*, is then given in terms of the interior volume and the cyclone height (Mothes and Löffler, 1984b).

$$R_w^* = \sqrt{\frac{\text{cyclone volume}}{\pi H}} = \left\{ \frac{1}{H} \left[\frac{(H-h)}{3} \left(R_w^2 + \frac{B^2}{4} + R_w \frac{B}{2} \right) + R_w^2 h \right] \right\}^{1/2}. \quad (1)$$

Formulation of the core region

Considering the particle concentration and wall roughness, Mothes and Löffler (1984a) obtained the tangential gas velocity profile in a modified cylinder as follows

$$v_\theta(R) = \frac{v_{\theta,w}}{\left(\dfrac{R}{R_w}\right)\left[1 + P\left(1 - \dfrac{R}{R_w}\right)\right]}. \quad (2)$$

In the equation $v_{\theta,w}$ is the tangential velocity at the edge of turbulent core near the wall, and is expressed by the wall friction coefficient ξ and friction-free velocity $v_{\theta,w}^*$.

$$v_{\theta,w} = \frac{v_d}{\xi h^*} \left[\sqrt{\frac{1}{4} + \xi h^* \frac{v_{\theta,w}^*}{v_d}} - \frac{1}{2} \right] \quad (3)$$

$$v_{\theta,w}^* = v_d \frac{\pi R_w^2}{ab[-0.204(b/R_w) + 0.889]} \quad (4)$$

$$v_d = \frac{Q}{\pi R_w^2} \quad (5)$$

$$h^* = \frac{a}{R_w} \left[\frac{2\pi - \cos^{-1}(b/R_w - 1)}{2\pi} - 1 \right] + \frac{h}{R_w} \quad (6)$$

$$P = \frac{v_{\theta,w}}{v_d} \left(\xi + \frac{\xi}{\sin \epsilon} \right) \quad (7)$$

$$\xi = 0.0065 \sim 0.0075, \quad (8)$$

where P is a parameter representing the momentum exchange between gas and the wall; Q is the volume flow rate; and ϵ the cone slope. Although the expression for v_θ looks quite complicated, the result agrees very well with the well-known velocity profile based on the power law, that is, $v_\theta \sim R^{-n}$, with the vortex exponent n given by an experimental correlation of Alexander (1949) and in the range of $0.5 \sim 0.9$. The advantage of this expression is that it gives a quantitative value of tangential velocity at the edge of the turbulent core, $v_{\theta,w}^*$, which has sometimes been assumed equal to the inlet velocity due to lack of information.

In analyzing particle motion in the turbulent-core region, a new coordinate system in terms of the radial coordinate R and the streamwise coordinate s along the swirling flow (Enliang and Yingmin, 1989) is more convenient than the conventional radial–circumferential coordinate (R, θ). Since the axial gas velocity is much lower than the tangential velocity, it is assumed that the streamwise velocity is equal to the tangential velocity in the new coordinates (R, s). When it is further assumed that particles slip only in the radial direction and particle motion is quasi steady, particle trajectories are simply expressed in the normalized radial coordinate $r(= R/R_w)$ based on the outer radius of the real cylinder body and the streamwise coordinate s:

$$\frac{R_w dr}{v_r} = \frac{ds}{v\theta}, \qquad (r_e < r < r_w^*, \quad 0 \le s \le s_{\max}) \quad (9)$$

where

$$v_r = \frac{\tau}{R_w} \frac{v_\theta^2}{r} \quad (10)$$

$$v_\theta = \frac{v_{\theta,w}}{r[1 + P(1-r)]} \quad (11)$$

$$r_e = \frac{D_e}{2R_w} \quad (12)$$

$$s_{\max} = 2\pi R_w^* \left(\frac{H - a/2}{a} \right) \quad (13)$$

$$\tau = \frac{\rho_p d_p^2 C_c}{18\mu} \quad (14)$$

$$C_c = 1 + \frac{2}{P_a d_\mu}[6.32 + 2.01\exp(-0.1095 P_a d_\mu)]. \quad (15)$$

In these formulas, s_{max} is the maximum flow length; τ is the particle relaxation time; C_c is the slip correction factor; P_a is the absolute pressure in cmHg; and d_μ is the particle diameter in μm. In deriving Eq. 9 the radial gas velocity is neglected. In real cyclones the inward radial gas velocity induces a radially inward drag force to prohibit particles from entering the near-wall region, but in the cylindrical cyclones the radial velocity has the effect of decreasing the streamwise velocity, giving more flow time for particle migration. Although it is obvious that these two effects work in opposite directions for particle collection, it is not clear whether the net effect is positive or negative. Hence, it is assumed in this study, after Enliang et al. (1989), that the two factors will almost cancel each other out. Then the radial gas velocity is neglected and the streamwise velocity is assumed unchanged along the streamwise coordinate s.

Integration of Eq. 9 gives the streamwise position s_1 where particles migrate to enter the near-wall region as a function of the radial position at the inlet (r_i), and also gives the innermost radial position at the inlet (r_i^*), which corresponds to r_w^* at the outlet, that is, $s_1(r_i^*) = s_{max}$:

$$s_1(r_i) = K_0\left[\frac{K_1}{3}(r_w^{*3} - r_i^3) - \frac{1}{4}(r_w^{*4} - r_i^4)\right] \quad (16)$$

$$s_{max} = K_0\left[\frac{K_1}{3}(r_w^{*3} - r_i^{*3}) - \frac{1}{4}(r_w^{*4} - r_i^{*4})\right] \quad (17)$$

$$K_0 = P\left(\frac{R_w^2}{\tau v_{\theta,w}}\right) \quad (18)$$

$$K_1 = \frac{1+P}{P}. \quad (19)$$

When only the turbulent-core region is considered, the collection efficiency will be determined by the particle flux entering the near-wall region,

$$\eta_{core} = \frac{\int_{r_i^*}^{r_w^*} u_i(r) c_i(r)\, dr}{\int_{r_e}^{r_w^*} u_i(r) c_i(r)\, dr}, \qquad (r_i^* < r_w^*). \quad (20)$$

Here u_i and c_i are the velocity and concentration of particles at the inlet. When the velocity and concentration is assumed uniform at the inlet, Eq. 20 can be further simplified like Eq. 21. This simplification looks practically reasonable, considering that at the inlet of real cyclones the tangential velocity is not yet developed to the power-law shape, so the inlet velocity at the tangential entrance is nearly uniform:

$$\eta_{core} = \frac{r_w^* - r_i^*}{r_w^* - r_e}, \qquad (r_i^* < r_w^*). \quad (21)$$

Formulation of the boundary-layer region

Exact velocity distribution inside the boundary layer is not known yet, either for conical or cylindrical cyclones, and it can be easily imagined that the boundary layer in cyclones will be different from the boundary layer over a flat plate or inside a circular tube without swirling motion. For all the apparent differences in flow conditions, the mean streamwise velocity distribution inside the near-wall region is assumed in this study by the law-of-the-wall on flat plate due to lack of specific information. Since the boundary layer is very thin throughout a cyclone, this assumption may be quite reasonable from the practical point of view, and potential errors caused by the assumed velocity profile together with other uncertainties will be corrected by adjusting the penetration depth, the only adjustable parameter in this study, to experimental data as in Eq. 38.

It is a common practice to express the velocity profile inside a boundary layer in a dimensionless form, based on the wall friction velocity u_*. The dimensionless velocity profile over a flat plate, $v_\theta^+ \ (= v_\theta/u_*)$ vs. the dimensionless distance from the wall, $y^+ (= yu_*/\nu)$, is

$$v_\theta^+(y^+) = \begin{cases} y^+ & y^+ \le 5 \\ a_0 + a_1 y^+ + a_2 y^{+2} + a_3 y^{+3} & 5 < y^+ \le 30 \\ 2.5\ln y^+ + 5.5 & 30 < y^+, \end{cases} \quad (22)$$

where $a_0 = -1.076$, $a_1 = 1.445$, $a_2 = -0.04885$, $a_3 = 0.0005813$ (Kallio and Reeks, 1989). The wall friction velocity u_* is related to the free stream velocity above the near-wall region and the friction factor f:

$$u_* = v_\theta(r_w^*)\sqrt{\frac{f}{8}}. \quad (23)$$

Although the friction factor is dependent on the wall roughness and flow structure, it is assumed here to be constant at 0.02 as in Barth (1956).

Since the centrifugal force in the turbulent core is much larger than in the near-wall region, most of the particles escaping the boundary layer back into the turbulent core will be returned back to the near-wall region. Then the interface between the turbulent core and the near-wall region plays the role of a reflecting surface, and the boundary-layer region can be considered to be a duct confined by a reflecting wall and a collecting wall, from the particles' point of view.

The collection probability of particles moving inside such a near-wall region can be expressed in terms of the deposition velocity V_d defined as the ratio of deposition flux to the average concentration. For a channel defined by average parameters such as mean flow velocity U, perimeter P_e, length of passage L, and flow cross-sectional area A, the efficiency can be expressed as an exponential function of only one deposition parameter (Kallio and Reeks, 1989). Errors arising from using mean flow velocity and so on will be handled by choosing a proper value for V_d through comparison with experiments:

$$\eta = 1 - \exp\left[-\frac{V_d P_e L}{UA} \right]. \tag{24}$$

The deposition velocity, V_d, consists of two components: $V_{d,t}$ by turbulent diffusion including Brownian diffusion, and $V_{d,c}$ by centrifugal force.

Wood (1981) suggested an empirical equation for dimensionless particle deposition velocity $V_{d,t}^+$ that is nondimensionalized by the turbulent wall friction velocity u_* on the rough surface in turbulent streams as

$$V_{d,t}^+ = \frac{1}{I_B + I_S} \tag{25}$$

$$I_B = \int_5^{30} \frac{\nu dy^+}{(D_b + D_t)_B} \tag{26}$$

$$= \begin{cases} 24.2 & y_{\text{off}}^+ \le 5 \\ 5 \ln\left(\dfrac{25.2}{y_{\text{off}}^+ - 4.80} \right) & y_{\text{off}}^+ > 5 \end{cases} \tag{27}$$

$$I_S = \int_{y_{\text{off}}^+}^{5} \frac{\nu dy^+}{(D_b + D_t)_S} \tag{28}$$

$$= \begin{cases} 14.5 Sc^{2/3} [f(\phi) + g(\phi) - f(\phi_1) - g(\phi_1)] & y_{\text{off}}^+ \le 5 \\ 0 & y_{\text{off}}^+ > 5 \end{cases} \tag{29}$$

$$y_{\text{off}}^+ = 0.45 k^+ + 0.69 \tau^+ \tag{30}$$

$$f(\phi) = \frac{1}{6} \ln \frac{(1+\phi)^2}{1 - \phi + \phi^2} \tag{31}$$

$$g(\phi) = \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2\phi - 1}{\sqrt{3}} \right) \tag{32}$$

$$\phi = 0.345 \mathrm{Sc}^{1/3} \tag{33}$$

$$\phi_1 = (0.041 k^+ + 0.0476 \tau^+) \mathrm{Sc}^{1/3}, \tag{34}$$

where D_b is the Brownian diffusivity of the particle, D_t is the turbulent eddy diffusivity; $k^+ (= k u_* / \nu)$ is the dimensionless roughness height; $\tau^+ (= \tau u_*^2 / \nu)$ is the dimensionless particle relaxation time; and Sc $(= \nu / D_b)$ is the Schmidt number. In Eqs. 26–34 subscripts S and B represent the viscous sublayer and buffer layer in a turbulent boundary layer, respectively, and the lower limit of integration y_{off}^+ was given empirically in order to consider the surface roughness and particle stopping distance.

As was mentioned previously, the additional deposition velocity by centrifugal force $V_{d,c}^+$ comes from the nonequilibrium particle motion for which the quasi-steady drag coefficient is not applicable, and so can be estimated only by directly integrating the particle motion with turbulent dispersion and centrifugal force considered simultaneously. Instead of simulating for a large number of particles, it assumed here that the deposition velocity, which is the average velocity of particles' hitting on the wall, corresponds to the equilibrium migration velocity at some point $y^+ = y_c^+$ inside the boundary layer:

$$V_{d,c}^+ \simeq \tau^+ \left. \frac{v_\theta^{+2}(y^+)}{R_w^{*+}} \right|_{y^+ = y_c^+}. \tag{35}$$

Using the notion of deposition velocity, the collection probability of a particle that enters the near-wall region at $s^+ = s_1^+$ is expressed as

$$\eta = 1 - \exp\left[-\frac{V_d^+}{U^+} \frac{(s_{\max}^+ - s_1^+)}{\delta^+} \right] \tag{36}$$

with

$$V_d^+ \simeq V_{d,t}^+ + \tau^+ \left. \frac{v_\theta^{+2}(y^+)}{R_w^{*+}} \right|_{y^+ = y_c^+}. \tag{37}$$

Strictly speaking, the thickness of the near-wall region δ^+ is more or less different from the boundary-layer thickness of gas flow in that the near-wall region here is defined as the region within which particles are collected with different migration velocity from that in the turbulent core. Therefore, δ^+ depends on the particle size and becomes zero for particles large enough to penetrate the boundary layer without much deceleration. Also the mean flow velocity U^+ in the near-wall region has to be chosen carefully, since it is related to the thickness of the near-wall region. Instead of trying hard to choose U^+ deliberately, the tangential velocity at the turbulent core edge, $v_\theta^*(r_w^*)$, is used for U^+ together with a fixed value of δ^+ for all particle sizes, and the errors caused by these simplifications are corrected by choosing a proper y_c^+.

If the motion of a particle inside the boundary layer is always in equilibrium with the surrounding gas flow, the migration velocity of the particle will monotonically decrease to zero as the particle approaches the wall. Therefore the deposition by centrifugal force, if any, should result from the nonequilibrium particle motion. That is, the actual migration velocity inside the boundary layer is different from and larger than equilibrium migration velocity based on the local flow velocity. Since the difference in the real migration velocity and the equilibrium migration velocity is increasing with particle inertia, y_c^+, which is a measure of the difference, should also be dependent on particle inertia. As a first approximation, y_c^+ is assumed here to be linearly proportional to the dimensionless particle relaxation time and bounded by the thickness of the near-wall region, and a value of 30 is used for δ^+, which is often used as boundary-layer thickness when particle motion is considered near a smooth wall (Li and Ahmadi, 1993):

$$y_c^+ \simeq \begin{cases} C_y \tau^+ \\ \delta^+ & \text{if} \quad C_y \tau^+ > \delta^+. \end{cases} \tag{38}$$

The proportional constant C_y is determined by comparing with the experimental data.

Consequently the collection efficiency of cyclones when the boundary-layer effects are considered is given by

$$\eta = \frac{1}{r_w^* - r_e} \int_{r_i^*}^{r_w^*} \left[1 - \exp\left\{ - \frac{V_d^+}{v_\theta^+(r_w^*)} \frac{(s_{\max}^+ - s_1^+)}{\delta^+} \right\} \right] dr. \quad (39)$$

Results and Discussion

The Stairmand high-efficiency cyclone (1951) with $D = 0.305$ m (Table 1) is considered here in order to facilitate an extensive comparison with available experimental data. The experimental data chosen for comparison are those of Dirgo and Leith (1985), where spherical liquid droplets of mineral oil ($\rho_p = 860$ kg/m^3) are used as particles so that they are not bounced or reentrained after striking the cyclone wall.

In the turbulent-core region, average particle trajectories are determined on the assumption of quasi-steady drag once initial starting positions are given. The position s_1 where a particle enters the near-wall region is a function of the starting position and the ratio of the radial migration velocity to the streamwise particle velocity, as shown in Figure 3, where $d_\mu^2 V_i$ represents the ratio of radial migration velocity to the streamwise velocity of a particle if other parameters such as cyclone geometry, gas properties, and particle density are fixed. Hence, the collection efficiency for the turbulent-core region alone, η_{core}, is represented by a single curve for various inlet gas velocities if plotted vs. $d_\mu^2 V_i$, but the experimental data show a substantial variation for different V_i, even when plotted vs. $d_\mu^2 V_i$ (Figure 4).

First of all, it is observed that all the experimental data points for the inlet velocity $V_i \leq 15$ m/s lie below the η_{core} curve. The discrepancy between the theoretical curve and the experiments, which becomes bigger for the lower inlet velocity, reflects the fact that the probability of particles reaching the near-wall region and that of reaching the wall are different. Since η_{core} is the probability of reaching the near-wall region with the reduction in efficiency by boundary-layer effects neglected, it is the upper bound for final efficiency, η. That is, η should always be smaller than η_{core}, which is well observed for $V_i \leq 15$ m/s. However, it is also observed that some experimental data show higher efficiencies than η_{core}, particularly when $V_i = 20$ and 25 m/s for intermediate-sized particles. Since η_{core} is calculated based on the quasi-steady drag assumption in the mean turbulent flow field, a higher efficiency than η_{core} is possible if the particle motion is not in equilibrium with the surrounding gas or turbulent diffusion enhances particle migration. As gas velocity increases the velocity gradient in the radial direction also increases. Then the dynamic characteristics of particle motion increases, and the real migration velocity of particles becomes higher than the equilibrium migration velocity. And errors in the experimental data are also suspected in that the experimental data do not show a smooth change of efficiency with the parameter $d_\mu^2 V_i$, but are abnormally high only for a particular size. It is not easy to judge at this stage what is the real phenomenon behind the abnormally high efficiencies, but it will be a meaningful work if the experimental data below the η_{core} curve can be analyzed satisfactorily.

For comparison, the results of previous theories are plotted in Figure 5, where the curve for the Barth model is for the modified Barth model where the ratio of settling velocity, v_{ts}/v_{ts}^*, was intentionally quadrupled for better fitting without any proper reasoning (Dirgo and Leith, 1985). First of all, it is observed that all the models shown predict much lower efficiencies than the experimental results. This is quite different from the predictions of the present model where η_{core}

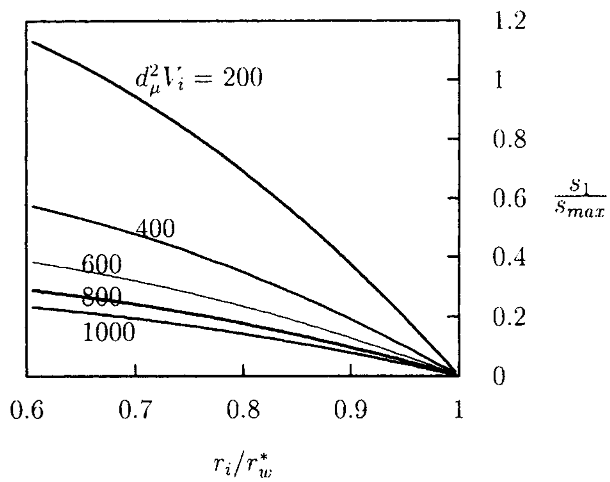

Figure 3. Relationship between the radial position at the inlet and the streamwise position entering the near-wall region.

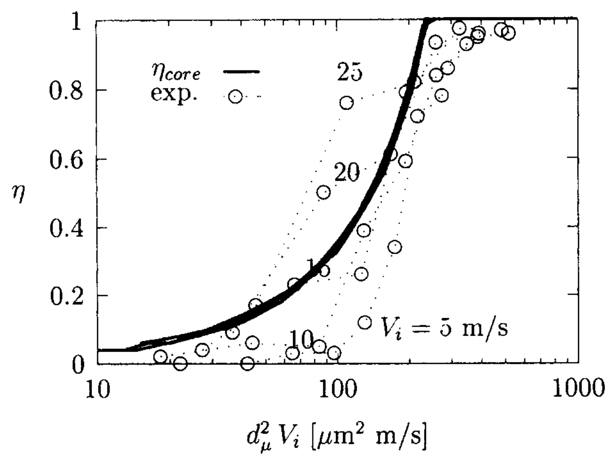

Figure 4. Collection efficiency for the turbulent core region alone vs. experimental data.

$V_i = 5$, 10, 15, 20 and 25 m/s.

Table 1. Dimensions of the Stairmand High-Efficiency Cyclone Design

Dimension	Dimension Ratio (Dimension/D)	Actual Length (m)
Cyclone diameter, D	1.000	0.305
Gas outlet diameter, D_e	0.500	0.152
Inlet height, a	0.500	0.152
Inlet width, b	0.200	0.061
Outlet duct length, S	0.500	0.152
Cylindrical body height, h	1.500	0.457
Cyclone height, H	4.000	1.220
Dust outlet diameter, B	0.375	0.114

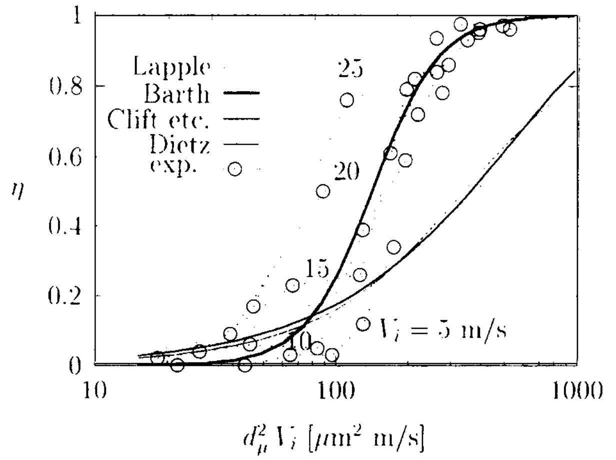

Figure 5. Existing models vs. experimental data (Dirgo and Leith, 1985).

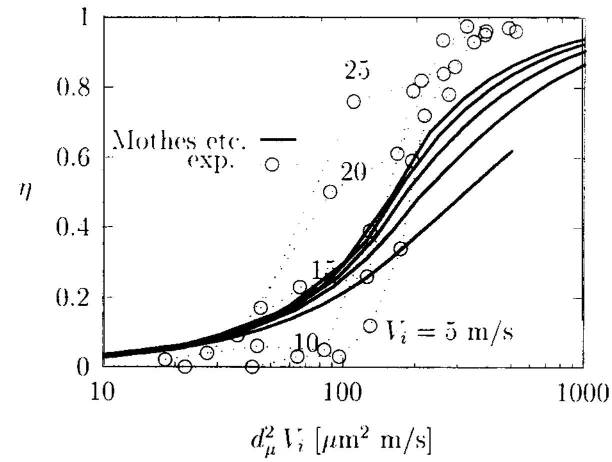

Figure 6. Mothes and Löffler's model ($D_p = 0.0125$) vs. experimental data (Dirgo and Leith, 1985).

correctly defines the upper bound for experimental results (Figure 4). The discrepancy comes from the different assumptions on radial mixing in the turbulent-core region. The increase in collection efficiency with particle size is due to the increased migration velocity. Since the particle flux on the wall is proportional to not only the migration velocity but also the particle concentration, proper account of the particle concentration has to be taken. As particle size increases, not only the migration velocity but also the particle concentration becomes large near the wall, with a boundary condition of nonzero concentration gradient (Mothes and Löffler, 1984a). Therefore increased particle size contributes to an increased collection efficiency through both an increased particle concentration and an increased migration velocity. When a complete mixing is assumed in the core region, the concentration at the wall is underestimated, so the collection efficiency increases slowly with particle size as in most of the previous theories. Since the degree of underestimation becomes larger for the larger particles, the estimated collection efficiency curve for the turbulent-core region becomes higher and stiffer than those of the previous theories, when turbulent dispersion is properly considered.

Unlike the other models, the Mothes and Löffler model shows some variation of collection efficiency with inlet gas velocity as a result of including the effective particle dispersion coefficient, D_p (Figure 6). However, the collection efficiency for large particles is still underestimated, which comes from the overestimation of mixing by turbulent dispersion. Although a reduced D_p could increase the collection efficiency, it at the same time makes the efficiency insensitive to the variation of gas velocity because it makes the model equivalent to the Dietz model in a conceptual sense.

When the near-wall region, which acts as a barrier to particle collection, is included, the particle that is considered to be collected in previous models has a collection probability of less than 1.0, because the exponent in Eq. 36, $-V_d^+(s_{\max}^+ - s_1^+)/(U^+\delta^+)$, is finite. As was previously mentioned, C_y has to be chosen properly because it is directly connected to the estimation of the deposition velocity. A value of 14 was chosen for C_y, by trial and error, and then the results of this study agree very well with the experimental data (Figure 7).

The relationship between y_c^+ and the particle size for the case of $C_y = 14$ is plotted in Figure 8, where we can see that y_c^+ reaches it upper bound for particles larger than a few micrometers. Although for those large particles the efficiency might be underestimated due to improper evaluation of boundary-layer effects (or y_c^+ and boundary layer thickness δ^+), the amount of underestimation will not be substantial because of the mathematical characteristics of the exponential function in Eq. 36 that the exponential function does not change much when the exponent is big. When the gas velocity is 5 m/s, y_c^+ does not reach its upper bound for the particle sizes considered here, and the good agreement with the experimental data proves that the boundary-layer effects are properly modeled.

Of the two components of deposition velocity, the component by turbulent diffusion, $V_{d,t}^+$ is smaller than that by centrifugal force, $V_{d,c}^+$ for a smooth surface and the conditions considered here (Figure 9). As is well known, $V_{d,t}^+$ is almost solely dependent on τ^+ except for small particles, which are affected more by Brownian diffusion than by their inertia, while $V_{d,c}^+$ depends on both τ^+ and V_i.

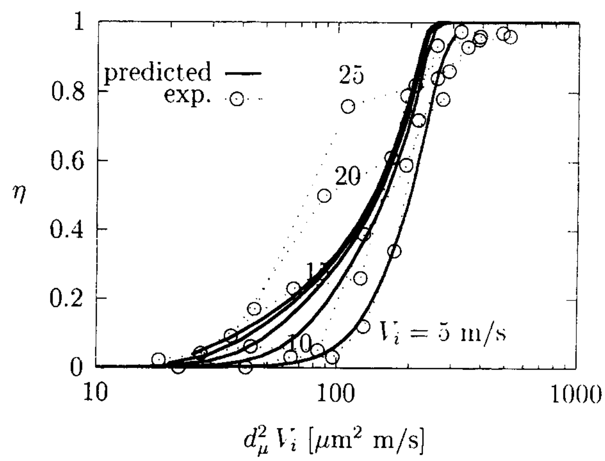

Figure 7. Results of this model agree very well with experimental data (Dirgo and Leith, 1985) $\delta^+ = 30$; $y_c^+ = 14\tau^+$).

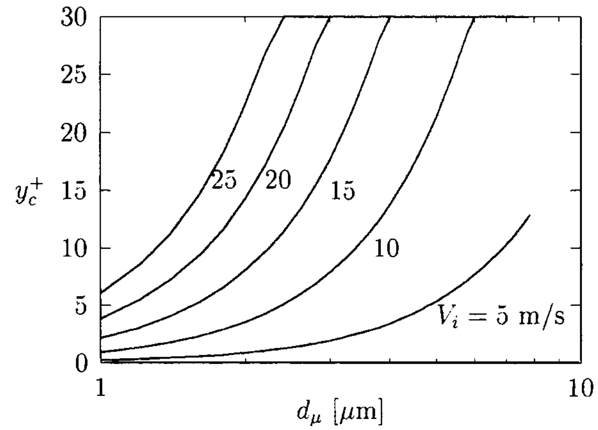

Figure 8. Relationship between y_c^+ and particle diameter for various inlet velocities ($\delta^+=30$; $y_c^+=14\tau^+$).

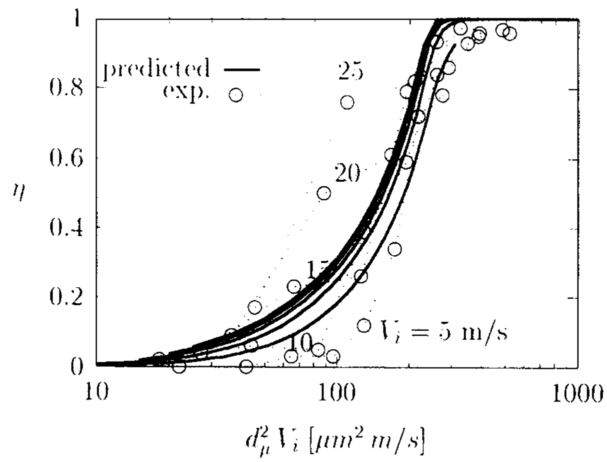

Figure 10. Efficiency predicted when y_c^+ is assumed constant and equal to 10 ($\delta^+=30$).

If y_c^+ is a constant rather than proportional to τ^+, the estimated efficiency curves for different V_i will become parallel with one another (Figure 10), then the wide spread of efficiencies for low-efficiency conditions cannot be properly predicted. It thus follows that y_c^+ should be varied depending on τ^+, and a linear relationship between y_c^+ and τ^+ will be one of good candidates.

Even though the specific calculation results shown are only for liquid droplets, the formulation in this article is really general, so that the deposition efficiency for solid particles can be reasonablly explained in the absence of a reentrainment effect if only the particle diameter used in the formulas is interpreted as the aerodynamic diameter. It thus follows that for particles of very high density y_c^+ becomes equal to δ^+ (Figure 8) at a much smaller size than for liquid droplets, and those particles with y_c^+ equal to δ^+ can be thought to be deposited once they reach the boundary layer. However, for solid particles of very high density the reentrainment or rebound becomes large, resulting in a reduced apparent deposition without rebound. Then C_y for solid particles, which is adjusted according to the observed deposition velocity, becomes smaller than for liquid droplets.

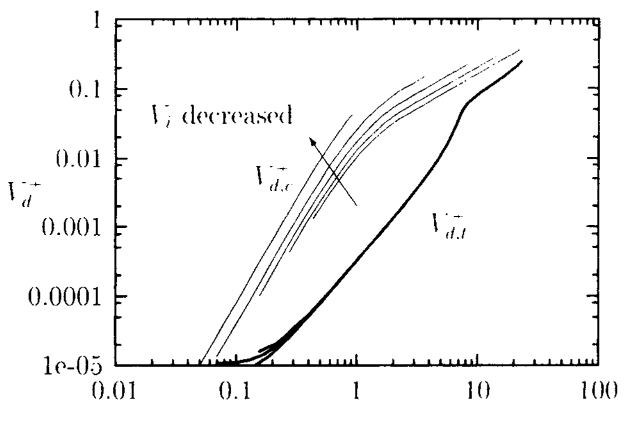

Figure 9. Typical values for the two components of deposition velocity on smooth surface.

$V_i = 5, 10, 15, 20, 25$ m/s; $\delta^+ = 30$; $y_c^+ = 14\tau^+$.

Even though the new model proposed here uses a simplified geometry and velocity field plus some incomplete simplifications, the results of the present model where the boundary layer effects are taken into consideration show an excellent enhancement in the prediction of collection efficiency for high-efficiency cyclones, which was impossible in any of the previous theories where the boundary-layer effect was totally neglected, and the importance of boundary layer-effects on particle deposition in cyclones is seen.

Conclusions

In the present article, a new efficient collection model for cyclones based on the characteristics of the wall boundary layer was developed. A cyclone is divided into the turbulent-core region and the near-wall region. The particle motion in the turbulent-core region is considered on the assumptions of quasi-steady drag and mean turbulent gas flow field in the simplified geometry. In the near-wall region, where the particle motion is not in equilibrium with the surrounding gas flow, particle flux on the wall is calculated by means of the deposition velocity, which consists of two terms, one by turbulent diffusion and the other by centrifugal force, which is assumed to be equal to the equilibrium migration velocity at a point, y_c^+, inside the boundary layer. y_c^+ is assumed to be linearly proportional to the dimensionless particle relaxation time, that is, $y_c^+ = C_y \tau^+$, and a value of 14 is chosen for C_y for good agreement with the experimental data.

Even though the new model proposed here uses a few incomplete simplifications about geometry, velocity field, and particle motion, the results of the present model where the boundary-layer effects are taken into consideration show an excellent enhancement in predicting the variation of collection efficiency with inlet velocity and particle size for high-efficiency cyclones, which was impossible in any of the previous theories where the boundary-layer effect was totally neglected, and the importance of boundary-layer effects on particle deposition in cyclones is seen.

Acknowledgment

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Notation

R_w = radius of the real cyclone ($= D/2$)
r = radial coordinate normalized with respect to R_w
r_e = normalized radius of the gas outlet ($= D_e/2R_w$)
T = absolute temperature [K]
v_r = radial velocity of particle
η = collection efficiency
ρ = gas density (1.18 kg/m^3)
ρ_p = particle density (860 kg/m^3)
μ = gas viscosity (1.835×10^{-5} kg/m·s)
ν = gas kinematic viscosity

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